Problema S100. Let ABC be an acute triangle with altitudes BE and CF. Points Q and R lie on segments CE and BF, respectively, such that $CQ/QE = FR/RB$. Determine the locus of the circumcenter of triangle AQR when Q and R vary.

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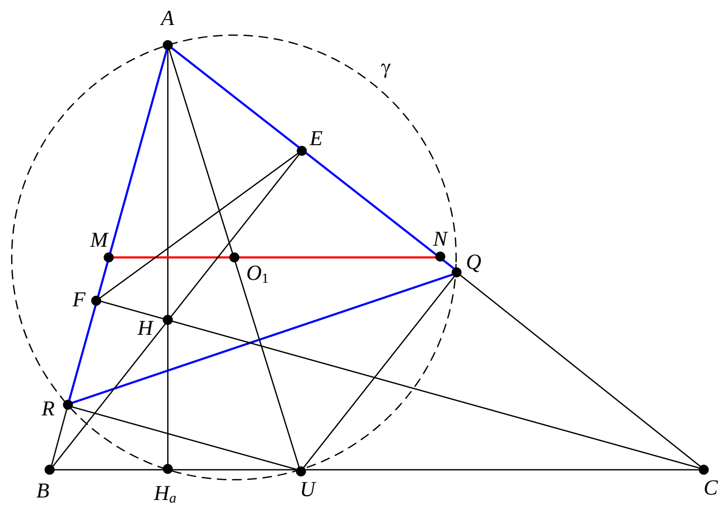

If Q is a point of CE, the point R can be costructed in the following way:

- through point Q draw a line parallel to BE to intersect BC at point U;
- through point U draw a line parallel to CF to intersect AB at point R;

From Thales' theorem we have:

$$\frac{CQ}{QE} = \frac{CU}{UB} \qquad , \qquad \frac{CU}{UB} = \frac{FR}{RB}$$

so the point R satisfies the relation:

$$\frac{CQ}{QE} = \frac{FR}{RB}$$

The circle γ with diameter AU contains H_a, Q, R because

$$\angle AQU = \angle ARU = \angle AH_aU = 90^\circ$$

Thus the circumcenter of $\triangle AQU$ is the mid-point O_1 of AU. This implies that the required locus is the set of mid-points of the cevians AU, where U is a variable point of BC. In other words the locus is the segment joining the mid-points M, N of the sides AB, AC. $\square$