Problema S101. Let a, b, c be distinct real numbers. Prove that

$$\left(\frac{a}{a-b}+1\right)^2+\left(\frac{b}{b-c}+1\right)^2+\left(\frac{c}{c-a}+1\right)^2\geq 5$$

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The proposed inequality follows from:

$$\left(\frac{a}{a-b}+1\right)^2+\left(\frac{b}{b-c}+1\right)^2+\left(\frac{c}{c-a}+1\right)^2-5=$$

$$=\frac{(2a^2b-ab^2-a^2c-3abc+2b^2c+2ac^2-bc^2)^2}{(a-b)^2(a-c)^2(b-c)^2}$$

□