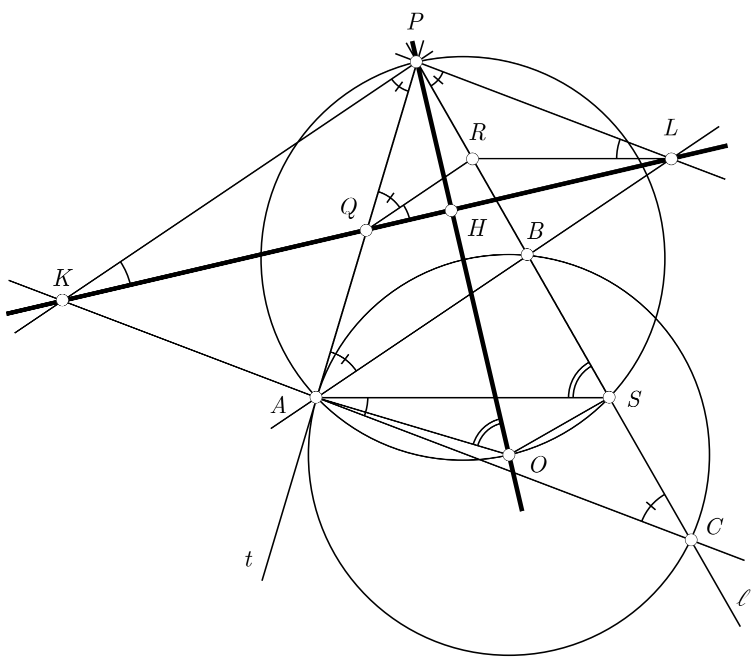
3439. *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

Let Γ be a circle with centre O and radius R. Line t is tangent to Γ at the point A, and P is a point on t distinct from A. The line ℓ distinct from t passes through P and intersects Γ at the points B and C. The point K is on the line AC and such that $PK \parallel AB$, and the point L is on the line AB and such that $PL \parallel AC$. Prove that $KL \perp OP$.

Solution by Mosca Sebastiano, Pescara, Italy and Ercole Suppa, Teramo, Italy



Let S the orthogonal projection of O onto BC, let R the midpoint of PB and let $Q = AP \cap KL$. Since $\angle OAP = \angle OSP = 90°$, the quadrilateral $AOSP$ is cyclic, so

$$\angle POA = \angle PSA \tag{1}$$

Since $ALPK$ is a parallelogram, Q is the midpoint of AP. Therefore $QR \parallel AB$ and this implies that

$$\angle KPA = \angle RQP = \angle LAP = \angle BCA = \angle CPL \tag{2}$$

Hence the triangles ABC and LBP are similar, so

$$\frac{AC}{PL} = \frac{AB}{BL} = \frac{BC}{PB} \qquad \Rightarrow \qquad \frac{AC}{PL} = \frac{CS}{PR} \tag{3}$$

From (3), taking into account of $\angle RPL = \angle ACS$, it follows that $\triangle RPL \sim \triangle SCA$ and this implies

$$\angle PLR = \angle CAS \tag{4}$$

Finally, by using (1), (2), (4) and external angle theorem's, we have

$$\begin{aligned}\angle HQP = \angle HQR + \angle RQP = \angle CAS + \angle BAP = \angle SCA + \angle SAC = \\ = \angle PSA = \angle POA = 90^\circ - \angle APO = 90^\circ - \angle QPH\end{aligned}$$

Therefore $\angle PQH + \angle QPH = 90^\circ$, so $KL \perp OP$ and the result is established. $\square$