**Problema J117.** Let $a$, $b$, $c$ be positive real numbers. Prove that

$$\frac{a}{2a^2+b^2+3}+\frac{b}{2b^2+c^2+3}+\frac{c}{2c^2+a^2+3}\le\frac{1}{2}$$

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Since $a^2+1\ge 2a$, $b^2+1\ge 2b$, $c^2+1\ge 2c$ we have

$$\sum_{\text{cyclic}}\frac{a}{2a^2+b^2+3}\le\sum_{\text{cyclic}}\frac{a}{4a+2b}$$

so we only need to prove that

$$\sum_{\text{cyclic}}\frac{a}{2a+b}\le 1$$

After easy computations the last inequality becomes

$$\frac{ab^2+bc^2+ca^2-3abc}{(2a+b)(2b+c)(2c+a)}\ge 0$$

which is true in virtue of **AM-GM** inequality

$$ab^2+bc^2+ca^2\ge 3\sqrt[3]{a^3b^3c^3}=3abc$$

Thus, the inequality is proven. The equality holds if and only if $a=b=c=1$. $\square$