**Problema J143.** Let $x_1 = -2$, $x_2 = -1$ and

$$x_{n+1} = \sqrt[3]{n\left(x_n^2 + 1\right) + 2x_{n-1}}$$

for $n \geq 2$. Find $x_{2009}$.

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We first notice that $x_3 = 0$, $x_4 = 1$, $x_5 = 2$, and so forth. This leads to the conjecture that $x_n = n - 3$. We prove this assumption by induction on $n$.

The claim is obvious for $n = 1$. Suppose that the assumption is proved for $k = 1, 2, \ldots, n$. Then by using the recurrence we have

$$\begin{aligned} x_{n+1}^3 &= n\left[(n-3)^2 + 1\right] + 2(n-4) = \\ &= n\left(n^2 - 6n + 10\right) + 2n - 8 = \\ &= n^3 - 6n^2 + 12n - 8 = (n-2)^3 \end{aligned}$$

Thus $x_n = n - 2$, hence the claim is valid for $n + 1$. Therefore $x_{2009} = 2006$. $\square$