Problema J151. Let $a \geq b \geq c > 0$. Prove that

$$(a-b+c)\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right) \geq 1$$

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Since $a \geq b \geq c > 0$, we have

$$\begin{aligned}
&(a-b+c)\left(\frac{1}{a}-\frac{1}{b}+\frac{1}{c}\right)-1= \\
=&\frac{(a-b+c)(bc-ac+ab)-abc}{abc}= \\
=&\frac{abc-a^2c+a^2b-b^2c+abc-ab^2+bc^2-ac^2+abc-abc}{abc}= \\
=&\frac{ab(a-b)-ac(a-b)+bc(a-b)-c^2(a-b)}{abc}= \\
=&\frac{(a-b)\left(ab-ac+bc-c^2\right)}{abc}= \\
=&\frac{(a-b)(b-c)(a+c)}{abc} \geq 0
\end{aligned}$$

which ends the proof. $\square$