

**Problema J163.** Let  $a, b, c$  be nonzero real numbers such that  $ab+bc+ca \geq 0$ .  
0. Prove that

$$\frac{ab}{a^2+b^2} + \frac{bc}{b^2+c^2} + \frac{ca}{c^2+a^2} \geq -\frac{1}{2}$$

*Proposed by Titu Andreescu, University of Texas at Dallas, USA*

*Solution by Ercole Suppa, Teramo, Italy*

We have obviously

$$\begin{aligned} \sum_{cyc} \frac{ab}{a^2+b^2} &= \sum_{cyc} \left( \frac{ab}{a^2+b^2} + \frac{1}{2} \right) - \frac{3}{2} \\ &= \sum_{cyc} \frac{(a+b)^2}{2(a^2+b^2)} - \frac{3}{2} \\ &\geq \sum_{cyc} \frac{(a+b)^2}{2(a^2+b^2+c^2)} - \frac{3}{2} \\ &= \frac{2(a^2+b^2+c^2) + 2(ab+bc+ca)}{2(a^2+b^2+c^2)} - \frac{3}{2} \\ &= 1 + \frac{ab+bc+ca}{a^2+b^2+c^2} - \frac{3}{2} \\ &= \frac{ab+bc+ca}{a^2+b^2+c^2} - \frac{1}{2} \geq -\frac{1}{2} \end{aligned}$$

where in the last step we have used the fact that  $ab+bc+ca \geq 0$ . □