Problema J170. In the interior of a regular pentagon $ABCDE$ consider the point M such that triangle MDE is equilateral. Find the angles of triangle AMB.

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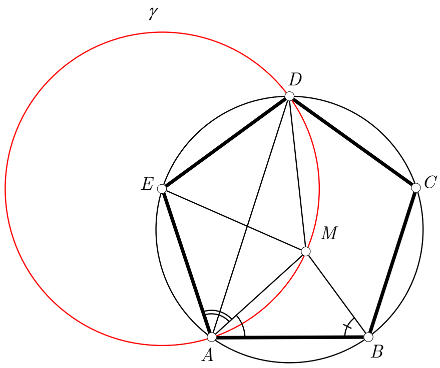
Notice that the points M, D, A lie on the circle γ with center E and radius ED. Furthermore M lie on the perpendicular bisector of DE so, clearly, $\angle MBA = \frac{1}{2}\angle ABC = 54°$.



Since MDE is equilateral we have $\angle DEM = 60°$. Therefore

$$\angle EAM = \angle EAD + \angle DAM = \frac{1}{2}\angle EMD + \frac{1}{2}\angle DEM = 36° + 30° = 66°$$
$$\angle MAB = \angle EAB - \angle EAM = 108° - 66° = 42°$$
$$\angle AMB = 180° - \angle MAB - \angle MBA = 180° - 42° - 54° = 84°$$

$\square$