Problema J171. If different letters represent different digits, could the addition

$$
\begin{array}{r}
AXXXU \\
BXXV \\
CXXY \\
+\ DEXXZ \\
\text{- - - - - - - -} \\
XXXXX
\end{array}
$$

be correct?

Proposed by Titu Andreescu, University of Texas at Dallas, USA

Solution by Ercole Suppa, Teramo, Italy

Considering the sum $\pmod 9$ we get

$$8X + (A + B + C + D + E + U + V + X + Y + Z) \equiv 5X \pmod 9 \qquad \Rightarrow$$
$$3X + 45 \equiv 0 \pmod 9 \qquad \Rightarrow \qquad 3X \equiv 0 \pmod 9$$

Therefore $X \in \{3, 6, 9\}$. We will prove that the only possible case is $X = 6$. To describe the our reasoning, we label the columns starting from the right.

Consider the three following cases

- If $X = 3$ we should have

$$
\begin{array}{r}
A333U \\
B33V \\
C33Y \\
+\ DE33Z \\
\text{- - - - - - - -} \\
33333
\end{array}
$$

  Notice that we must have a carryover of 1 from the first column. Adding the terms of the fourth column we get $B+C+D+4 = 13$ or $B+C+D+4 = 23$ (since B, C, D are different and $B, C, D \leq 9$).

  - If $B + C + E + 4 = 13$ then $A + D + 1 = 3$ which is impossible;
  - If $B + C + E + 4 = 23$ then $A + D + 2 = 3$ which is impossible.

- If $X = 9$ we should have

$$
\begin{array}{r}
A999U \\
B99V \\
C99Y \\
+\ DE99Z \\
\text{- - - - - - - -} \\
99999
\end{array}
$$

  Notice that we must have a carryover of 3 from the first column, i.e. $U + V + Y + Z = 39$, which is clearly impossible.

Thus the only possibility is $X = 6$ which gives the solution

$$
\begin{array}{r}
16662 \\
5667 \\
3668 \\
+\ 40669 \\
\text{- - - - - - - -} \\
66666
\end{array}
$$

Obviously we obtain other solutions by permuting the digits U, V, Y, Z in the first column, the digits B, C, E in the fourth column and the digits A, D in the fifth column. $\square$