Problema J239. Let a and b be real numbers so that $2a^2 + 3ab + 2b^2 \leq 7$. Prove that

$$\max(2a + b, 2b + a) \leq 4$$

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Because of symmetry we can assume, without loss of generality that $a \geq b$, so $\max(2a + b, 2b + a) = 2a + b$. Clearly we have

$$\begin{aligned}(2a + b)^2 &\leq (2a + b)^2 + 3b^2 + 2 = \\ &= 4a^2 + 6ab + 4b^2 + 2 = \\ &= 2\left(2a^2 + 3ab + 2b^2\right) + 2 \leq \\ &= 2 \cdot 7 + 2 = 16\end{aligned}$$

Therefore $2a + b \leq 4$ and we are done. $\square$