Problema O147. Let H be the orthocenter of an acute triangle ABC, and let A', B', C' be the midpoints of sides BC, CA, AB. Denote by A_1 and A_2 the intersections of circle $C(A', A'H)$ with side BC. In the same way we define points B_1, B_2 and C_1, C_2, respectively. Prove that points A_1, A_2, B_1, B_2, C_1, C_2 are concyclic.

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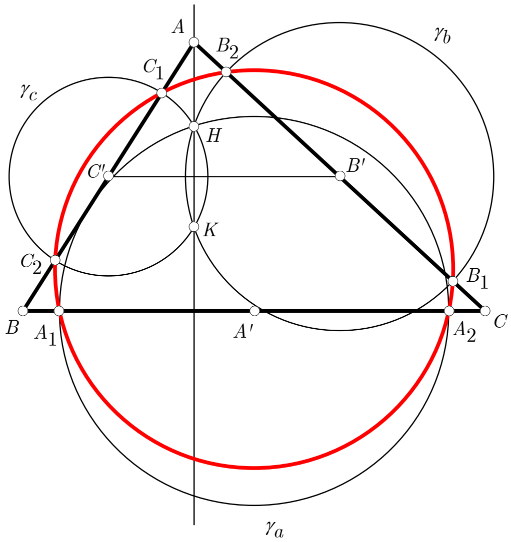

Let γ_a, γ_b, γ_c be the circles with centers A', B', C' and radii $A'H$, $B'H$, $C'H$ respectively. Denote by K the second intersection point of γ_b and γ_c, besides H.

Since $B'C' \parallel BC$ and $AH \perp BC$ we have $AH \perp B'C'$. Thus AH is the radical axis of γ_b and γ_c, so $K \in AH$. The power of a point theorem implies

$$AB_1 \cdot AB_2 = AH \cdot AK = AC_1 \cdot AC_2$$

so the points B_1, B_2, C_1, C_2 are concyclic. Since the axes of B_1B_2 and C_1C_2 intersect at the circumcenter O of $\triangle ABC$, it follows that B_1, B_2, C_1, C_2 lies on the circle with center O and radius OB_1.

Similarly, we can prove that B_1, B_2, A_1, A_2 lies on the circle with center O and radius OB_1. Therefore A_1, A_2, B_1, B_2, C_1, C_2 are concyclic and the proof is complete. □